

The preparation of data for the solution of simultaneous equations and matrix inversion by DEUCE.

by

P. C. Birchall.

Introduction.

A speedy service in the solution of simultaneous equations and matrix inversion is available and can be guaranteed if the data is correctly presented. This note is intended to serve you as a guide in the preparation of such data and if its rules are strictly observed you will get your results in the shortest possible time.

Data Preparation.

Data is punched by the customer on Hollerith cards. Fig. 1 is a picture of one such card with its twelve rows and eighty columns. Punching is a very simple operation. The card is placed in the punch feed and the carriage moved transversely until the chosen column is under the punch knives. The depression of the appropriate key then gives a punching in the required row, and the carriage is automatically stepped along to bring the next column of the card beneath the knives.

As the orders of the equations and matrices increase the layout of the punching changes. In all there are three types and these are described below.

Punches are available in the Maths. Service Dept., where a demonstration can be arranged. Cards are supplied on request.

Simultaneous Equations.

Let the number of equations be n and suppose that for the same coefficients of the unknowns, commonly called 'left hand sides' solutions are required for r different sets of 'right hand sides'.

1. First number the rows of the right hand sides and both rows and columns of the left hand sides from 1 to n , and number the columns of the right hand sides from $(n + 1)$ to $(n + r)$.
2. Then calculate the sums of $(n + r)$ elements in each of the n rows and assign to the column so created the number $(n+r+1)$.
3. Consult Fig. 2. The region in which (r, n) lies will tell you which type of punching is required.
4. For greatest accuracy all elements are scaled i.e., each row is multiplied by a power of ten so that the largest element excluding the row sums, lies in the interval $6 \times 10^7 \leq x < 6 \times 10^8$. Should any of the sums be too large to punch scale down all the elements in those rows until the offending sums are within capacity.
5. Form a further row $(n + 1)$ by taking the sums of the $(n + r)$ columns.
6. Punch the scaled elements including row and column sums: altogether $(n + 1)(n + r + 1)$ cards should be prepared.

The labelling of the rows and columns relating to a 3×3 matrix with 2 right hand sides is given below:

Col. No.	1	2	3	4	5	6.
Row No.						
1.	a_{11}	a_{12}	a_{13}	b_{11}	b_{12}	$\sum_{j=1}^3 a_{1j} + \sum_{s=1}^2 b_{1s} = a_{11} + a_{12} + a_{13} + b_{11} + b_{12}$
2.	a_{21}	a_{22}	a_{23}	b_{21}	b_{22}	$\sum_{j=1}^3 a_{2j} + \sum_{s=1}^2 b_{2s} = a_{21} + a_{22} + a_{23} + b_{21} + b_{22}$
3.	a_{31}	a_{32}	a_{33}	b_{31}	b_{32}	$\sum_{j=1}^3 a_{3j} + \sum_{s=1}^2 b_{3s} = a_{31} + a_{32} + a_{33} + b_{31} + b_{32}$
4.	$\sum_{i=1}^3 a_{i1}$	$\sum_{i=1}^3 a_{i2}$	$\sum_{i=1}^3 a_{i3}$	$\sum_{i=1}^3 b_{i1}$	$\sum_{i=1}^3 b_{i2}$	$\sum_{ij} a_{ij} + \sum_{is} b_{is}$

a_{ij} ($1 \leq i \leq 3$)
($1 \leq j \leq 3$) are the left hand sides.

b_{is} ($1 \leq i \leq 3$)
($1 \leq s \leq 2$) are the right hand sides.

If the elements of the right hand sides differ much in magnitude from those of the left hand sides it is advisable to scale them by a power of ten to bring them in line, before following the above procedure. The corresponding results must of course be rescaled accordingly.

Matrix Inversion.

Let the order of the matrix be n .

1. Number the rows and columns of the matrix from 1 to n .
2. Calculate the row sums and give them column number ($n + 1$)
3. Find what type of punching is needed.
For $n \leq 10$ it is type B;
For $10 < n \leq 15$ it is type A;
For $15 < n \leq 47$ it is type C, unless most of the elements are zero, when type A is used.
4. Scale all the elements, including sums by a power of 10 so that they are as large as the punching allows, subject to any restriction peculiar to the type of punching.
5. Calculate column sums and give them row number ($n+1$).
6. Punch all elements including row and column sums.

Colour Convention.

For ease in distinguishing between matrix elements, row sums etc., a Hollerith card colour convention is used:

Elements of matrices and left hand sides of equations are punched on buff cards;

right hand sides of equations on yellow stripe cards;
row sums on blue stripe cards;
column sums on grey stripe cards;
grand sums on salmon stripe cards.

Decimal Punching.

The rows and columns of a Hollerith card are labelled as shown in Fig.1. A hole punched in the Y row represents a positive sign whilst one in the X row

3.

a negative sign. Holes punched in rows 0,1,2....9 represent digits of that magnitude.

When punching cards it is essential that the following rules are observed:

- (I) All cards must be punched in every column specified whether the numbers are non significant zeros or not.
- (II) Only one hole may be punched in any column.
- (III) All cards must bear row and column numbers and be in order row by row.
- (IV) Row sums must always be exact; so that if in any scaling nonzero digits have been lost sums must be corrected accordingly.
- (V) The most significant digits of column sums which over-spill are not punched.
- (VI) All damaged cards are copied and then discarded.

Type A.

The sign of the number is punched in column 21 and ten decimal digits in columns 22-31. The row and column numbers of the element are punched in columns 35-38 and 42-45 respectively. Zero elements, unless they are sums, are not punched. Sums are punched with sign in column 21 and fourteen digits in columns 22-35. Row and column numbers are punched in columns 4-7 and 10-13 respectively. Remember that non-significant zeros are punched e.g. 0001 is punched for row or column 1.

Type B.

The sign of the number is punched in column 21 and ten decimal digits in columns 22-31. Numbers should be less than 1.07×10^9 when read as integers. Row numbers are punched in columns 4-7 and column numbers in 10-13. Row sums have a punching in the X row of column 17, column sums a punching in the X row of column 18 and the grand sum, which is the sum of row sums, a punching in both these positions. Row sums are punched in the same manner as the elements.

Type C.

Three numbers are punched on each card, with signs in 21, 31 and 41 and digits in columns 22-30, 32-40, and 42-50, each number having nine digits. Row numbers are punched in columns 4-7 and card numbers within the row in columns 10-13. If the number of elements, excluding the sum is not a multiple of three it is made so by adding zero elements. Row sums are punched on separate cards with sign in column 21 and fourteen digits in columns 22-35. Row and column number are punched as for other elements. Column sums are not punched but need to be supplied.

Mathematical Services Dept., R.A.E.,

19th June, 1956.

M.S. 2784

