NS t 1005

NELSON RESEARCH LABORATORIES
STAFFORD E.E.CO. LTD.

DEUCE Programme No. 12 (LE1).

Linear Equations (Non Magnetics).

SUMMARY

This report contains a description, operating instructions, flow diagram and coding for a DEUCE programme for solving linear equations and inverting a matrix without the use of the magnetic drum. The maximum number of equations which can be solved is 14 with 2 right hand sides and the largest matrix that can be inverted is a 10 x 10 matrix. In general if there are n equations with m right hand sides, n, m must satisfy the inequality $n(n+m+1) < 244$.

The programme is a literal translation of an ACE programme prepared by the N.P.L. and no attempt has been made to make the maximum use of the extra DEUCE facilities, although use has been made of these facilities where savings in time or storage space were clearly evident.

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DEUCE Programme No. 12. (LE1)

Linear Equations (Non Magnetics)

OPERATORS INSTRUCTIONS

Data Card

Y row nP17 (n is the number of equations).

X row mP17 (m is the number of right hand sides and is zero if matrix inversion only is required).

0 row P17 (If matrix inversion is required).

Order of Cards Programme cards 0-15; Data Card; Matrix; Programme cards 16-54.

Decimal Cards Are punched in columns 2 to 11 with sign in column 1. The cards should be sorted into rows and exact row sums punched on blue cards which are placed after the cards for the row. The maximum modulus of any number or row sum should be less than 2^{30} (approx. 1.07×10^9).

Output Answers are punched out decimally in columns with the inverse first if this is required. The final card punched is -1×10^P. This gives an indication of the position of the decimal point regarding the initial data as integers (10^0 is used in the diagonal unit matrix.)

Failure Indications

4	14 - 28	Read failure.
2	13 - 28	Row sum failure, initial input.
2	24 - 27	Numbers getting too large to reduce.
4	13 - 27	Arithmetic failure in reduce.
2	13 - 28	Storage failure during Squash up.

2. DESCRIPTION OF PROGRAMME

The programme will solve n linear simultaneous equations with m right hand sides where n, m must satisfy the inequality $n(n+m+1) \leq 244$. Thus, for example, 14 equations with 2 right hand sides or 7 equations with 26 right hand sides can be solved. The programme can also be used to invert a matrix, the programme itself replacing the right hand sides by the unit matrix. Therefore in this case n = m and we must have $n(2n+1) \leq 244$ so that the largest matrix that can be inverted is a 10 x 10.

The numerical method used is the familiar method of successive elimination of the variables followed by a back substitution process. Coefficients are punched one to a card as integers and the cards are sorted into rows and each row followed by a card bearing the exact row sum. The answers are punched out in decimal, one to a card, column by column the inverse of the matrix of the coefficients of the left hand sides being punched out first if this is required. The final card punched is -1 x 10P. This gives an indication of the position of the decimal point and is so arranged that the maximum possible accuracy is obtained in the results.

TRANSLATION OF ACE PROGRAMME

As a programme for solving linear equations was required quickly the obvious course was to translate the existing ACE Pilot Model programme. In order to do this with the minimum of programme testing on the DEUCE the following procedure was adopted. A programmer familiar with both ACE and DEUCE coding made a literal translation of the ACE flow diagram, instruction by instruction, making use of the extra DEUCE facilities where this could be done without any major rearrangement of the programme. Wherever possible the same storage locations were used in the DEUCE programme as in the original ACE programme and any change either in the form or location of the instruction was noted. When the flow diagram was complete it was then possible for an assistant, with sufficient knowledge of the DEUCE code, to carry out the purely mechanical process of coding the programme, again making a note of the instructions which were not literal translations of the corresponding ACE instructions. Guide cards were then prepared for use with the 'Interpreter' programme (DEUCE programme Z01) and a machine translation of the ACE programme was then made. By checking the punching of the cards obtained from this machine translation against the coding carried out by hand it was possible to eliminate sources of error due either to the programmer's incorrect translation of an ACE instruction, incorrect coding or a machine error during the operation of the 'Interpreter' programme. The instructions which did not correspond to instructions in the ACE programme were then punched and the programme was ready for testing. When the programme was tested the most likely source of error was this group of instructions and of the four errors which did, in fact, occur, three were due to faults in this group of instructions.

The main differences between the DEUCE and ACE programmes were the saving of instructions due to the continuous read and punch facilities of the DEUCE, the 1's carry into the odd half of DS21 when a negative number is added to the even half, the subtraction facility associated with DS21 and the use of the automatic divider instead of programmed division in calculating $2^{60-y}/10^{x}$. The ACE programme used DL1-3 and 11 for storing instructions and DL4-10 for coefficients. The DEUCE programme uses DL1-4 for instructions and DL5-12 for coefficients so that it is possible to solve more equations with the DEUCE programme than with the ACE programme because of the existence of the extra delay line.

3. ORGANISATION OF PROGRAMME

The programme consists of five parts:

(i)	Decimal read.	Cards 1-15
(ii)	Reduce	Cards 16-24
(iii)	Squash up for Back Substitution	Cards 25-30
(iv)	Back substitution	Cards 31-42
(v)	Decimal output	Cards 43-54

A P54 is punched on the 9's row of the last card of parts 2 to 5.

A number of failure indications are incorporated in the programme. In the Decimal read part of the programme there are failure indications if a card is not punched correctly or if there is a row sum failure. In its present form the failure indications do not stop the reader although they stop further operation of the machine. A slight amount of recoding would be necessary to make the reader stop at each of these failure indications.

During the reduction of the matrix there is a danger that one or more of the coefficients of the reduced matrix will be greater than single length. There is therefore a failure indication if the numbers become too large in the reduction and in addition there is an indication if there is an arithmetical failure as shown by the comparison of the sum of the reduced elements of one row with the reduced row sum.

A further failure indication is in the Squash up for Back Substitution part in which the coefficients of the reduced matrix are stored away in new positions. This indication occurs if the elements have not been stored away correctly.

4. INSTRUCTIONS FOR USE

Data Card Y row nP17

X row mP17 (m=0 for matrix inversion only).

0 row P17 if matrix inversion is required.

Order of Cards

Programme cards 0-15; Data Card; Matrix; Programme cards 16-54.

Decimal Cards

Are punched in columns 2 to 11 with sign in column 1. The cards should be sorted into rows and exact row sums punched on blue cards which are placed after the cards for the row. The maximum modulus of any number or row sum should be less than 2^{30} (approx. 1.07×10^9).

Output

Answers are punched out decimally in columns with the inverse first if this is required. The final card punched is -1×10^{P}. This gives an indication of the position of the decimal point regarding the initial data as integers (10^8 is used in the diagonal unit matrix.)

MATHEMATICAL PHYSICS LABORATORY

Failure Indications

4	14 - 28	Read failure.
2	13 - 28	Row sum failure, initial input.
2	24 - 27	Numbers getting too large to reduce.
4	13 - 27	Arithmetic failure in reduce.
2	13 - 28	Storage failure during Squash up.

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NELSON RESEARCH LABORATORIES
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DEUCE Programme No. 26 (LE02)

Linear Equations (Magnetics) $n + m \leq 31$

SUMMARY

The report contains a description, working instructions, flow diagram and coding for a DEUCE programme for solving a set of simultaneous linear algebraic equations, storing the elements in the magnetic drum store. The largest set of equations than can be solved is 30 x 30 with 1 right hand side.

The greater part of the programme, with the exception of the input routines "Read three decimal elements per card" and "Read binary elements", is a literal translation of an ACE programme prepared by the N.P.L. and the maximum use has not been made of the extra facilities available on DEUCE, although when there were obvious savings in time or storage space to be gained modifications to the original were made accordingly.

The input routines "Read three decimal elements per card" and "Read binary elements" were prepared and tested at N.R.L. Blackheath.

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DEUCE Programme No. 26 (LE02)

Linear Equations (Magnetics) $n + m \leq 31$

INSTRUCTIONS FOR USE

Cards 1000 - 1015 Decimal input; 1 element per card, sign and 10 digits.

Cards 1100 - 1121 Decimal input; 3 elements per card, sign and 9 digits each.

Cards 1200 - 1212 Binary input; 12 elements per card.

Cards 2001 - 2012 Reduce.

Cards 4001 - 4018 Back substitution. Decimal Output.

Cards 4101 - 4115 Back substitution. Binary Output.

Cards 5001 - 5003 Check Memory.

Sections 40, 41, 50 can be used or omitted as desired.
Section 50 should be used last.

Order of Cards

Input section. Parameter card. Data. Reduce section. Back sub. and output section. Check memory, (if desired).

Punching of Elements

In all input routines the elements of the right hand side of the equations should be punched consecutively with the elements of the left hand side, so it is not in general possible to punch left hand side elements and right hand side elements on different coloured cards, except when the 1 element per card input routine is used.

Cards 1000 - 1015

Decimal input. All elements. 1 element per card, sign in column 1 and 10 decimal digits in columns 2 - 11.

Programme will check against a row of column sums if desired.

A card for the grand sum should then be provided, but as long as it is decimally punched it doesn't matter what is on it.

No number (excluding row sums) should exceed 1.07×10^9. Suggest largest number between 6×10^7 and 6×10^8.

Parameter Card

Y - row. nP_{17} $n + m \leq 31$

X - row. mP_{17}

0 - row. P_{17} if row of column sums is included. Otherwise blank.

Failures

2	9 - 24	Card not decimally punched.
5	13 - 29	Row sum wrong (Difference on O.S.)
3	13 - 29	Column sum wrong (Difference on O.S.)

Cards 1100 - 1121

Decimal input. All elements. 3 elements per card, signs in columns 1, 11, 21, and 9 decimal digits each in columns 2-10, 12-20, 22-30.

The last card of each row should have three decimal numbers punched on it, the surplus numbers being punched as zeros.

The row sum cards have a sign in column 1, and 14 decimal digits in columns 2-15.

Limits as for section 10.

Parameter Card

Y - row. nP_{17} $n + m \leq 31$

X - row. mP_{17}

Failures

2	9 - 24	Element card not decimally punched.
5	9 - 24	Row sum card not decimally punched.
7	9 - 24	Row sum wrong. (Difference in DS21.)

Cards 1200 - 1212

Binary input. All elements. 12 elements per card.

The row sum should be punched immediatley after the last element of each row and may be shifted down if desired.

No number (excluding row sums) should exceed 2^{30}. Suggest largest number between 2^{26} and 2^{29}.

Parameter Card

Y - row nP_{17} $n + m \leq 31$

X - row mP_{17}

0 - row kP_{17} where row sums divided by 2^k are to be read in ($0 \leq k \leq 16$).

Failure

4	9 - 24	Row sum wrong.

Cards 2001 - 2012

Reduce.

Failures

4	30 - 29	Arithmetic failure. (Difference in TS13)
2	30 - 29	Numbers getting too big. (Number in TS14).

Cards 4001 - 4018

Back substitution. Decimal output. -10^D is punched on a card preceding the solution for each R.H.S. then D decimal places are given, assuming original data as integers.

Cards 4101 - 4115

Back substitution. Binary output. -2^p is punched on a separate card preceding each solution then p binary places are given, assuming original data as integers. Each solution will have a sum divided by 2^5.

Cards 5001 - 5003

Check memory.

Failure

2 26 - 28

(i) P_{17} is in TS13; then track (i) is at fault.