DEUCE Programme No. 730 (XR03/1).

Multiple Regression Programme.

SUMMARY.

A DEUCE Program to fit by least squares a linear form to a set of values of a set of n variables, using the General Interpretive Program. By means of a simple code, q dependent variables and p independent variables can be selected from any of the n variables. By a similar code, those results which are optional can be selected. Limitations on the numbers of variables are $n \leq 45$, $p \leq 30$ and $q \leq 12$.

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## ORGANISATION OF PROGRAMME.

Reads an input parameter card specifying type of data and output required.

Reads data of Type I or II or III and calculates results 1-6, as required for all variables. Reads a calculation parameter card specifying p and q and the output required (if different from that specified on the input parameter card). Calculates results 7-14 as required for this selection of variables p, q then returns to read a new calculation parameter card.

### Data.

#### Type I.

Decimal Data. An Mxn matrix of M readings of n variables, punched with 12, 5 digit numbers per card with 3 decimal places. Spilt row sums are optional, as also are zeros in blank columns.

The data is punched as for Subroutine R29T, preceded by a binary parameter card with

overpunched signs
17-21, 22-26, ...., 47-51, ...

M x P17 on the Y row

n x P17 on the X row

where M = number of readings

n = number of variables

also with P1 on the 2 row if there are zeros in final columns of a row up to column 76.

P17 on the 2 row if spilt row sums are included.

#### Type II.

Triangle of double length sums of squares and cross-products punched in binary. This will usually have been obtained from a previous run of the program as Output 1.

#### Type III.

Three matrices (the number of readings, covariances and means) punched in binary, usually obtained as outputs 2, 3 and 4 from a previous run of the program.

### Results.

These are punched in binary (64 column) and then converted to decimal with 6 9-digit numbers per card.

(1) A triangle of double length elements of sums of squares and cross products bordered at the left by the sums of the values of each variable. This may be used as Type II input.

(2) * M number of readings.
(3) * Covariance matrix (nxn).
(4) * Means (nxl).

} This may be used as Type III input.

(5) Simple regression Coefficients.

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ORGANISATION OF PROGRAMME CONT'D.

(6) Simple correlation coefficients.

(7) Inverse of independent covariance.

(8) Correlation between regression coefficients.

(9) * Regression coefficients (A).

(10) * Regression constants.

(11) Multiple correlation coefficients.

(12) Residual variances for dependent variables ($\sigma^2$)

(13) Standard errors of regression coefficients ($\sigma$)

(14) 't' values $= \frac{A}{\sigma}$

* The results marked thus are always punched.

Time.

The greatest part of the time is usually taken up in reading the decimal data and forming the sums of squares and cross products. This time is given very approximately as $\frac{Mn\ (n + 1)\ \text{mins}}{9600}$. The additional time for the calculation varies from about 5 mins for very small cases to the order of 40 mins for n = 30 and all the optional output for one selection of p, q.

OPERATING INSTRUCTIONS.

Input.

(1) Programme pack (to card 273).

(2) Input/output Parameter card, with:

(a) Type of input: 1, 2 or 3 x P1 on Y row.

(Inputs Types II, III are output 1 and outputs 2, 3, 4).

(b) Selection of Output: P1......P14 as required on X row.

(3) Set of data, possible forms are described above.

(4) Calculation parameter cards with:

(a) Either Y row blank, or a selection of P7......P14, as required. If the Y row is blank, the output is that specified on the Input/output parameter card.

(b) X row and 0 row. Punch to correspond to the selection of q dependent variables up to 12 out of up to 45 original variables. For example, to select the 3rd variable, the 35th variable and the 36th variable punch P3 on the X row and P3 and P4 on the 0 row.

(c) 1 Row and 2 Row. Select p independent variables in a similar way to the dependent variables ($p \leq 30$)

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## OPERATING INSTRUCTIONS CONT'D.

### Output.

Outputs 2, 3, 4 will always be punched, and outputs 9 and 10 every time a selection of dependent and independent variables is made. A digit punched on output selection parameter card on the X row will give equivalent output i.e. A P7 will give output 7. The binary matrices are punched with a label x P5 on the 9 row indicating the output number.

### Notes.

The program will read only one set of data as the bricks to form cross-products and covariance are overwritten. To repeat for another set read in the program again.

The program LK11T is suggested as a suitable method of conversion from binary to decimal.

### Programme Pack.

| Brick No. | Name. | Card Nos. |
|---|---|---|
| | GIP 8T | 0 - 33 |
| | Parameter card with 29 P17 | 34 |
| 1. | Codewords (DL's 5-3 codewords, DL2 unit matrix, DL 1,10 as matrix) | 35 - 50 |
| 2. | Square root LZ18B/1 | 51 - 57 |
| 3. | Flatten Matrix LZ30B/3 | 58 - 61 |
| 4. 5. 6. | Matrix Multiplication LM08B | 62 - 86 |
| 7. | Extract Diagonal LZ11B | 87 - 93 |
| 8. | Extract general submatrix LZ44B | 94 - 103 |
| 9. | Punch Binary Matrix LP15BT | 104 - 107 |
| 10. | Prepare for reduce LJ04B | 108 - 114 |
| 11. 12. | Reduce LJ01Z | 115 - 127 |
| 13. | Back substitute LB04B | 128 - 137 |
| 14. | Check store LC04Z | 138 - 141 |
| 15. | Binary Read LR16BT | 142 - 148 |
| 16. | Extract parameters LZ47B | 149 - 152 |
| 17. 18. | Term by Term matrix Arithmetic LZ61BM/1 | 153 - 177 |
| 19. | Select Element LZ12B | 178 - 181 |
| 20. | Form symmetric matrix LN03B | 182 - 191 |
| 21. | Truncate LZ60B | 192 - 198 |
| 22. 23. 24. | Form covariance triangle XR02B | 199 - 223 |

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| | | |
|---|---|---|
| 25. 26. | Extract sub-trapezoid LN05B (double length) | 224 - 239 |
| 27. 28. 29. | Form triangle of sums of squares and cross products 6L318/1 (XR01B/1) | 240 - 264 |
| | Codewords - 3 triads | 265 - 273 |

July, 1962.

2L139 R.M. Hatley, L.C.S.

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DEUCE GIP CODEWORD SHEET $O_{0-31}$

Card Nos. 1000 – 1002

| Ins. Lo. | a | b | c | r | | |
|---|---|---|---|---|---|---|
| 0 Input Line Kept Sieve | | 1 | 1 | 47 | Read Codewords | 0 |
| 1 Sieve | | $1_0$ | 2 | 47 | Read Codewords | 1 |
| 2 | Marker = P.1 Initially | | | | | 2 |
| 3 | 14 – 31 | | | D | Track 14/8 | 3 |
| 4 | 8 – 30 | | | D | | 4 |
| 5 | | | $1_{31}$ | 33 | | 5 |
| 6 | 14 – 31 | | | D | Track 14/6 | 6 |
| 7 | 6 – 30 | | | D | | 7 |
| 8 | | | $1_{31}$ | 33 | | 8 |
| 9 | 3/5 | (12) | (32) | 9 | Punch $\bar{\sigma}^2$ (q×1) | 9 |
| 10 | $O_1$ | Z $O_{22}$ | NZ $O_{11}$ | 35 | Sieve clear? | 10 |
| 11 | 9/10 | (1) | | 3 | Flatten diagonal of $\overline{M}C^{-1}$ (p×1) | 11 |
| 12 | 3/4 | 9/10 | 3/6 | 4 | $\frac{\bar{\sigma}^2}{M}$. diagonal $C^{-1}$ (q×p) | 12 |
| 13 | 3/6 | | 2/1 | 2 | Square Root | 13 |
| 14 | $1_6$ | $1_{10}$ | $O_{15}$ | 46 | Shift up and clear for $\sigma$ | 14 |
| 15 | 111 | Z $O_{17}$ | NZ $O_{16}$ | 35 | $\sigma$ ? | 15 |
| 16 | 2/1 | (13) | (32) | 9 | Punch $\sigma$ (q×p) | 16 |
| 17 | $1_6$ | $1_{10}$ | $O_{18}$ | 46 | Shift up and clear for 't' | 17 |
| 18 | 111 | Z $O_{22}$ | NZ $O_{19}$ | 35 | 't' ? | 18 |
| 19 | 4 | | | 48 | Divide $B/\sigma$ | 19 |
| 20 | 4/8 | 2/1 | 4/0 | 17 | | 20 |
| 21 | 4/0 | (14) | (32) | 9 | Punch t (q×p) | 21 |
| 22 | $O_0$ | | $O_1$ | 40 | Reset sieve | 22 |
| 23 | $O_{27}$ | | $O_2$ | 40 | Reset marker. | 23 |
| 24 | $O_3$ | $1_{31}$ | $2_{10}$ | 46 | Jump to read new card. | 24 |
| 25 | | | | | | 25 |
| 26 | | | | | | 26 |
| 27 | P.7 | | | | Marker | 27 |
| 28 $\gamma_1$ | 15 – 15X | | | | | 28 |
| 29 $\gamma_2$ | | | $1_0$ | 33 | | 29 |
| 30 $P_1$ | 2P1 | | | | | 30 |
| 31 $P_2$ | 3P1 | | | | | 31 |

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DEUCE GIP CODEWORD SHEET $1_0 - 31$

Card Nos. 1003 - 5

| mc. no. | a | b | c | r | | |
|---|---|---|---|---|---|---|
| 0 | 12 - 24 | | | D | | 0 |
| 1 | $0 - 9_0$ X | | | D X | Input type | 1 |
| 2 | $0 - 9_1$ X | | | D X | Output parameter | 2 |
| 3 | 9 - 24 | | | D | | 3 |
| 4 | 121 | $0_0$ | $1_{25}$ | 37 | Type I input ? | 4 |
| 5 | | | 0/0 | 27 | Read and form X-products (n+1)(n+2) | 5 |
| 6 | $9_2$ - 14 | | | D | Shift | 6 |
| 7 | 24 - $9_2$ | | | D | | 7 |
| 8 | $9_1$ - 15 | | | D | | 8 |
| 9 | 25 - 14 | | | D | Clear and examine whether required | 9 |
| 10 | 26 - $9_1$ | | | D | or not | 10 |
| 11 | 111 | $\overset{Z}{1_{13}}$ | $\overset{NZ}{1_{12}}$ | 35 | Punch X - Products ? | 11 |
| 12 | 0/0 | (1) | (32) | 9 | Punch X - Products (n+1)(n+2) | 12 |
| 13 | 0/0 | 0 | 1 | 48 | Extract M (1×2) | 13 |
| 14 | 0 | 1 | 6/13 | 25 | | 14 |
| 15 | 0/0 | 1 | 0 | 48 | Extract Means (n×2) | 15 |
| 16 | 0 | 1 | 6/10 | 25 | | 16 |
| 17 | 0/0 | 1 | 0 | 48 | Extract X - Products n(n+2) | 17 |
| 18 | 1 | 0 | 0/0 | 25 | | 18 |
| 19 | 6/13 | | 5/13 | 21 | Truncate $M \times 10^6$ | 19 |
| 20 | 5/13 | | | 19 | Select element | 20 |
| 21 | 0/0 | 6/10 | 0/0 | 22 | Covariance n(n+1) | 21 |
| 22 | 4 | | | 48 | $\frac{M \times 10^6}{10^6}$ | 22 |
| 23 | 5/13 | 14/4 | 14/4 | 17 | | 23 |
| 24 | $1_6$ | $1_{10}$ | $2_0$ | 46 | Shift up and clear to P3, jump | 24 |
| 25 | $0_{30}$ | $0_0$ | $2_{14}$ | 37 | Type II Input ? | 25 |
| 26 | | (1) | 0/0 | 15 | Read X - products | 26 |
| 27 | $1_6$ | $1_{10}$ | $1_{13}$ | 46 | Shift up and clear to P2, jump | 27 |
| 28 | P5 | | | | | 28 |
| 29 | 14 - 31 | | | D | | 29 |
| 30 | 7 - 30 | | | D | | 30 |
| 31 | 15 - 31 | | | D | | 31 |

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DEUCE GIP CODEWORD SHEET $2_{0-31}$

Card Nos. 1006 - 8

| m.c. no. | a | b | c | r | | |
|---|---|---|---|---|---|---|
| 0 | 14/4 | (2) | (32) | 9 | Punch M | 0 |
| 1 | 0/0 | | 0/0 | 21 | Truncate Covariance $\frac{n(n+1)}{2}$ | 1 |
| 2 | 6/10 | | 6/8 | 21 | Truncate Means $(n \times 1)$ | 2 |
| 3 | 4 | | | 48 | Reciprocal of $M \times 10^6$ | 3 |
| 4 | 14/5 | 5/13 | 5/13 | 17 | | 4 |
| 5 | 3 | | | 48 | Mult. Means $\times \frac{1}{M \times 10^6}$ | 5 |
| 6 | 6/8 | 5/13 | 5/5 | 17 | | 6 |
| 7 | 0/0 | 5/13 | 0/0 | 17 | Mult Covariance $\times \left(\frac{1}{M \times 10^6}\right)^2$ | 7 |
| 8 | 0/0 | 5/13 | 0/0 | 17 | " | 8 |
| 9 | 0/0 | | 5/7 | 20 | Form symmetric Covariance $(n \times n)$ | 9 |
| 10 | $1_6$ | $1_{10}$ | $2_{11}$ | 46 | Shift and clear to P4, jump | 10 |
| 11 | 5/7 | (3) | (32) | 9 | Punch covariance C $(n \times n)$ | 11 |
| 12 | 5/5 | (4) | (32) | 9 | Punch Means Y $(n \times 1)$ | 12 |
| 13 | $1_6$ | $1_{10}$ | $2_{21}$ | 46 | Shift and clear to P5, jump | 13 |
| 14 | $0_{31}$ | $0_0$ | $0_{28}$ | 37 | Type III Input ? | 14 |
| 15 | | (2) | 14/4 | 15 | Read M | 15 |
| 16 | | (3) | 5/7 | 15 | Read Covariance C | 16 |
| 17 | | (4) | 5/5 | 15 | Read Means Y | 17 |
| 18 | $1_{28}$ | | $0_2$ | 40 | Set marker to P5 | 18 |
| 19 | $11_{31}-14$ | | | D | Clear sieve to P5, jump. | 19 |
| 20 | $1_8$ | $1_{10}$ | $2_{21}$ | 46 | | 20 |
| 21 | 5/7 | | 0/0 | 7 | Extract diagonal of covariance $(1 \times n)$ | 21 |
| 22 | 4 | | | 48 | Reciprocal | 22 |
| 23 | 14/5 | 0/0 | 0/2 | 17 | | 23 |
| 24 | 0/2 | (1) | | 3 | Flatten $(n \times 1)$ | 24 |
| 25 | $1_6$ | $1_{10}$ | $2_{26}$ | 46 | Shift and clear to P6 | 25 |
| 26 | 111 | Z $0_3$ | NZ $2_{27}$ | 35 | Simple Regression ? | 26 |
| 27 | 3 | | | 48 | Pre-mult for simple regression $(n \times n)$ | 27 |
| 28 | 5/7 | 0/2 | 0/4 | 17 | | 28 |
| 29 | 0/4 | (5) | (32) | 9 | Punch simple regression $(n \times n)$ | 29 |
| 30 | | | $0_3$ | 33 | Jump to 14/8 | 30 |
| 31 | $P(1-\frac{1}{2})$ | | | | | 31 |

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DEUCE GIP CODEWORD SHEET 14/8 dl 5

Card Nos. 0001 - 3

| mo. nc. | a | b | c | r | | |
|---|---|---|---|---|---|---|
| 0 | $1_6$ | $1_{10}$ | $2_1$ | 46 | Shift up for correlation | 0 |
| 1 | 111 | Z $2_9$ | NZ $2_2$ | 35 | Simple correlation ? | 1 |
| 2 | 0/2 | | 0/2 | 2 | Square root (mx1 ~~nxn~~) | 2 |
| 3 | 3 | | | 48 | 'Pre - Mult' | 3 |
| 4 | 5/7 | 0/2 | 0/4 | 17 | | 4 |
| 5 | 0/2 | | | 3 | Flatten (1xN) | 5 |
| 6 | 3 | | | 48 | 'Post Mult' | 6 |
| 7 | 0/4 | 0/2 | 0/4 | 17 | | 7 |
| 8 | 0/4 | (6) | (32) | 9 | Punch simple correlation (nxn) | 8 |
| 9 | $0_1$ | | $0_0$ | 40 | Keep Sieve | 9 |
| 10 | 12 - 24 | | | D | Read Parameter Card | 10 |
| 11 | $0 - 1_{10}$ X | | | DX | Y row Output parameters | 11 |
| 12 | $0 - 9_{28}$ X | | | DX | X row $q_1$ } dependent | 12 |
| 13 | $0 - 9_{29}$ X | | | DX | 0 row $q_2$ | 13 |
| 14 | $0 - 9_{30}$ X | | | DX | 1 row $p_1$ } independent. | 14 |
| 15 | $0 - 9_{31}$ X | | | DX | 2 row $p_2$ | 15 |
| 16 | 9 - 24 | | | D | | 16 |
| 17 | $2_0$ | Z $2_{19}$ | NZ $2_{18}$ | 35 | New sieve ? for output types | 17 |
| 18 | $2_0$ | | $0_1$ | 40 | Replace sieve | 18 |
| 19 | 30 | | 96 | 40 | $p_1$ $12_0$ | 19 |
| 20 | 31 | | 97 | 40 | $p_2$ $12_1$ Extract ππ C (pxp) | 20 |
| 21 | 30 | | 104 | 40 | $p_3$ $12_8$ col | 21 |
| 22 | 31 | | 105 | 40 | $p_4$ $12_9$ | 22 |
| 23 | 5/7 | | 1/15 | 8 | ~~from s.s. & x products~~ covariance | 23 |
| 24 | $2_{19}$ | $2_{20}$ | $2_{25}$ | 46 | | 24 |
| 25 | $31 - 12_8$ | | | D | ~~store $10_4$ col~~ Extract π Y (px1) independant means | 25 |
| 26 | 5/5 | | 9/7 | 8 | Means | 26 |
| 27 | 1/15 | | | 10 | | 27 |
| 28 | | | | 11 | ππ $C^{-1}$ | 28 |
| 29 | | | 1/15 | 13 | | 29 |
| 30 | | | | 14 | | 30 |
| 31 | | | $1_{29}$ | 33 | Jump to 14/17 | 31 |

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DEUCE GIP CODEWORD SHEET 14/7 $d\ell$ 4

Card Nos. 0004 - 6

| mc. no. | a | b | c | r | | |
|---|---|---|---|---|---|---|
| 0 | $1_6$ | $1_{10}$ | $2_1$ | 46 | Shift up and clear for $\pi\pi C^{-1}$ | 0 |
| 1 | 111 | $2_3^{Z}$ | $2_2^{NZ}$ | 35 | Punch $\pi\pi C^{-1}$ ? | 1 |
| 2 | 1/15 | (7) | (32) | 9 | Punch $\pi\pi C^{-1}$ $(p\times p)$ | 2 |
| 3 | 1/15 | | 9/10 | 7 | Extract diagonal of $\pi\pi C^{-1}$ $(1\times p)$ | 3 |
| 4 | $1_6$ | $1_{10}$ | $2_5$ | 46 | Shift up and clear for correlation between reg. | 4 |
| 5 | 111 | $2_{15}^{Z}$ | $2_6^{NZ}$ | 35 | Correlation between regression coefficients? | 5 |
| 6 | 4 | | | 48 | $\frac{1}{\text{diag. of } \pi\pi C^{-1}}$ $(1\times p)$ | 6 |
| 7 | 14/5 | 9/10 | 3/12 | 17 | | 7 |
| 8 | 3/12 | | 3/12 | 2 | $\sqrt{(\text{diag } \pi\pi C^{-1})^{-1}}$ $(1\times p)$ | 8 |
| 9 | 3 | | | 48 | Pre-mult | 9 |
| 10 | 3/12 | 1/15 | 0/0 | 17 | | 10 |
| 11 | 3/12 | (1) | | 3 | Flatten $(p\times 1)$ | 11 |
| 12 | 3 | | | 48 | Post-mult | 12 |
| 13 | 3/12 | 0/0 | 0/0 | 17 | | 13 |
| 14 | 0/0 | (8) | (32) | 9 | Punch correlation between regression coeffs. $(p\times p)$ | 14 |
| 15 | 28 | | 96 | 40 | $q_1$ | 15 |
| 16 | 29 | | 97 | 40 | $q_2$ | 16 |
| 17 | 28 | | 104 | 40 | $q_1$ Extract $xx$ $C(q\times q)$ | 17 |
| 18 | 29 | | 105 | 40 | $q_2$ | 18 |
| 19 | 5/7 | | 1/10 | 8 | from covariance | 19 |
| 20 | $2_{15}$ | $2_{16}$ | $2_{21}$ | 46 | | 20 |
| 21 | 30 | | 104 | 40 | $p_1$ Extract $x\pi$ $C$ $(q\times p)$ | 21 |
| 22 | 31 | | 105 | 40 | $p_2$ | 22 |
| 23 | 5/7 | | 3/12 | 8 | from covariance | 23 |
| 24 | 3/12 | 1/15 | 4/8 | 4 | Matrix mult $A = x\pi C . \pi\pi C^{-1}$ | 24 |
| 25 | $2_{15}$ | $2_{16}$ | $2_{26}$ | 46 | | 25 |
| 26 | 31 - 128 | | | D | Extract $xY$ $(q\times 1)$ | 26 |
| 27 | 5/5 | | 9/9 | 8 | Dependent means. | 27 |
| 28 | $1_6$ | $1_{10}$ | $2_{29}$ | 46 | Shift up and clear for A | 28 |
| 29 | 4/8 | (9) | (32) | 9 | Punch regression coefficients A $(q\times p)$ | 29 |
| 30 | 9/7 | | | 3 | Flatten $\pi Y$ $(1\times p)$ [See also line 25] | 30 |
| 31 | | | $0_6$ | 33 | Jump to 14/6. | 31 |

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DEUCE GIP CODEWORD SHEET 14/6 d13

Card Nos. 0007 - 9

| mc. no. | a | b | c | r | | |
|---|---|---|---|---|---|---|
| 0 | 9/7 | | 1/15 | 16 | Extract parameters (o, m, n, p) | 0 |
| 1 | 1/15 | 2 | | 19 | Select element ("n") | 1 |
| 2 | 1 | | | 48 | Add; p+1 | 2 |
| 3 | −1 | 14/5 | 1/15 | 17 | | 3 |
| 4 | 4/8 | 9/7 | 2/0 | 4 | Matrix mult $A_\pi Y$ (q×1) | 4 |
| 5 | 2 | | | 48 | Subtract $xY - A_\pi Y$ | 5 |
| 6 | 9/9 | 2/0 | 2/1 | 17 | | 6 |
| 7 | $1_6$ | $1_{10}$ | $2_8$ | 46 | | 7 |
| 8 | 2/1 | (10) | (32) | 9 | Punch K (q×1) | 8 |
| 9 | 4/8 | 3/12 | 2/0 | 4 | Matrix Mult $A_\pi \times C$ (q×q) | 9 |
| 10 | $0_1$ | $0_{22}^{Z}$ | $2_{11}^{NZ}$ | 35 | Sieve clear? | 10 |
| 11 | $1_6$ | $1_{10}$ | $2_{12}$ | 46 | Shift up and clear for mult. correlation | 11 |
| 12 | 1/1 | $2_{19}^{Z}$ | $2_{13}^{NZ}$ | 35 | Multiple correlation? | 12 |
| 13 | 2/0 | | 3/0 | 7 | Extract diagonal $A_\pi \times C$ | 13 |
| 14 | 1/10 | | 3/1 | 7 | Extract diagonal $x \times C$. | 14 |
| 15 | 4 | | | 48 | Divide | 15 |
| 16 | 3/0 | 3/1 | 3/2 | 17 | | 16 |
| 17 | 3/2 | | 3/0 | 2 | Square Root | 17 |
| 18 | 3/0 | (11) | (32) | 9 | Punch multiple correlation coeffs. (1×q) | 18 |
| 19 | $0_1$ | $0_{22}^{Z}$ | $2_{20}^{NZ}$ | 35 | Sieve clear? | 19 |
| 20 | 2 | | | 48 | $x \times C - x_\pi C \cdot A$ (q×q) | 20 |
| 21 | 1/10 | 2/0 | 2/0 | 17 | | 21 |
| 22 | 14/4 | 1/15 | 1/15 | 17 | $M - (p+1)$ | 22 |
| 23 | 2/0 | | 3/3 | 7 | Extract diagonal (1×q) $(x \times C - x\pi C \cdot A)$ | 23 |
| 24 | 3/3 | (1) | | 3 | Flatten (q×1) | 24 |
| 25 | 4 | | | 48 | Divide $\frac{1}{M-p-1}$ . diag. $(x \times C - C \cdot A)$ | 25 |
| 26 | 3/3 | 1/15 | 3/4 | 17 | | 26 |
| 27 | $1_6$ | $1_{10}$ | $2_{28}$ | 46 | Shift up and clear for $\bar{\sigma}^2$ | 27 |
| 28 | 1/1 | $0_{10}^{Z}$ | $2_{29}^{NZ}$ | 35 | $\bar{\sigma}^2$ ? | 28 |
| 29 | 3 | | | 48 | Mult. $\frac{M}{M-p-1}$ . diag. $(x \times C - CA)$ | 29 |
| 30 | 3/4 | 14/4 | 3/5 | 17 | | 30 |
| 31 | | | $0_9$ | 33 | | 31 |