S.B.43

THE ENGLISH ELECTRIC CO., LTD.

NELSON RESEARCH LABORATORIES
STAFFORD
MATHEMATICAL PHYSICS LABORATORY.

Telephone :—Stafford 700.

Report No. NS t 113
Date 7.3.56.
Reference
Order No.

Front Sheet.
Data Sheet 1
Figure Sheets. S6/10408,9,10.

DEUCE Subroutine No. 123. (A08C)

Report by

N.P.L.

SUMMARY.

The attached document contains details of a DEUCE Subroutine which has been prepared and tested by "N.P.L."

MATHEMATICAL PHYSICS LABORATORY.

HEF.

NELSON RESEARCH LABORATORIES
STAFFORD E. E. CO. LTD.

Description. This subroutine performs arithmetical operations on complex numbers. The link is placed in 1_0 for Multiply Divide and Modulus, and in 1_1 for Square Root.

Data. $x = 2^p\,(a_1+ib_1)$, $y = 2^p\,(a_2+ib_2)$ $\quad 17 \leqslant p \leqslant 31$

Result. $c = 2^p\,(a_3+ib_3)$ for MULTIPLY, DIVIDE and SQUARE ROOT.

$c = 2^p\sqrt{a_1^2+b_1^2}$ for MODULUS

For MULTIPLY $\quad a_3=a_1a_2 - b_1b_2,\ b_3=a_1b_2+a_2b_1 \qquad (c = xy)$

For DIVIDE $\quad a_3 = \dfrac{a_1a_2 + b_1b_2}{a_1^2 + b_1^2},\ b_3 = \dfrac{a_1b_2-a_2b_1}{a_1^2+b_1^2} \qquad (c = \dfrac{y}{x})$

For SQUARE ROOT $\quad a_3= \left[\dfrac{a_1+\sqrt{a_1^2+b_1^2}}{2}\right]^{\frac{1}{2}},\ b_3 = \dfrac{b_1}{2a_3} \qquad a_1 \geqslant 0$

$(c = \sqrt{x})$

$a_3= \dfrac{b_1}{2b_3},\ b_3= \left[\dfrac{|a_1| + \sqrt{a_1^2 + b_1^2}}{2}\right]^{\frac{1}{2}} \qquad a_1 < 0$

For MODULUS $\quad c = |x|$

Uses F.2 (43) in D.L.$5_{1-25,28,29,31}$; D13 (68) in D.L.$6_{0,3,5-29}$ and M01/1 (26) in D.L.$2_{20-25,27-31}$.

Instructions for use.

Stores used.	13	14	15	16	17	19_2	19_3	20	21_2	21_3
Contents at entry.	link	-	-	-	-	2^pb_1	2^pa_1	-	2^pb_2	2^pa_2
Contents at exit	*	-	-	-	-	2^pb_1 *	2^pa_1	-	2^pb_3	2^pa_3

*N.B. After DIVIDE 19_2 contains -2^pb_1; After MODULUS 13 contains $c = 2^P \quad a_1^2 + b_1^2$

Occupies. 2,3,4_{0-17}, $5_{1-25,28,29,31}$ $6_{0,3,5-29}$.

Entry. MULTIPLY 2_{26}, DIVIDE 3_{23}, SQUARE ROOT 3_{26}, MODULUS 3_{27}.

Parameters. Wait No. (2p-33) in 2_{11}. Wait No. (2p-40) in 3_4, 4_{14} and 4_{16}.

NOTE: If SQUARE ROOT is not required D.L.'s 4 and 5 and 2_{18}, $3_{18-20,25-27}$ may be omitted. MULTIPLY incorporates a 2nd order subroutine with entry in 2_{19}. The contents at entry are as for MULTIPLY. At exit 20 contains $2^{2p}\,(a_1a_2 - b_1b_2)$ and 21 contains $2^{2p}\,(a_2b_1 + a_1b_2)$ 19 is unaltered. This subroutine uses 13,14,15,16,$17_{2,3}$, 19,20 and 21.

MODULUS and SQUARE ROOT incorporate a 2nd order subroutine with entry in 3_{28}. The contents at entry are as for modulus. At exit 21 contains $2^{2p}(a_1{}^2 + b_1{}^2)$ 19 is unaltered. This subroutine uses 13,14,16,19,20 and 21.

MULTIPLY

2_{26} 13 − 1_0
2_0 2_{11} − 13 [3 21-22 1 (2p-53) 30]
2_{19} $\boxed{\text{S.R.1}}$
$1_{31}(2_{11}$ 21 − 22)(64−2p mc). $2^{32+p}(a_2b_1+a_1b_2)$
3_{31} 29 − 23_2
3_2 21_3 − 15
2_5 20 − 21(2mc) $2^{2p}(a_1a_2-b_1b_2)$
2_{11} 21 22 (64-2p mc) $2^{32+p}(a_1a_2-b_1b_2)$
3_{11} 29 − 23_2
2_{16} 15 − 21_2
↓
1_0 Link

MODULUS

3_{27} 13 − 1_0
3_{18} 3_{20} − 13 [5 3-13 18 3]
3_{28} $\boxed{\text{S.R.2}}$
$1_{31}(3_{20}$ 3_{19} − 13) [1 13-13 0 0]
5_{14} $\boxed{\text{F.2}}$
$1_{30}(3_{19}$ 13 − 13)
↓
1_0 Link

S.R.1

2_{19} 13 − 1_{31}
2_8 21_2 − 16 $2^p b_2$
2_{12} 2_{14} − 13 [2 21-20 2 0 1]
2_{15} 21_3 − 15 $2^p a_2$
2_{17} 19_2 − 14 $2^p b_1$
2_{20} $\boxed{\text{M01/1}}$
$1_{30}(2_{14}$ 21 − 20) (2mc) $2^{2p} b_1 b_2$
2_1 2_{14} − 13 [2 21-17 2 2 3].
2_{13} 19_3 − 14 $2^p a_1$
2_{20} $\boxed{\text{M01/1}}$
$1_{30}(2_4$ 21 − $17_{2,3}$)(2mc) $2^{2p} a_1 b_2$

2_3 15 − 16 $2^p a_2$
2_6 2_{10} − 13 [2 20-23 2 0 2]
2_{20} $\boxed{\text{M01/1}}$
$1_{30}(2_{10}$ 20 − 23)(2mc) $2^{2p}(a_1a_2-b_1b_2)$
2_2 2_7 − 13 [1 17-22 2 2 31]
2_{14} 21 − 20(2mc) $2^{2p}(a_1a_2-b_1b_2)$
2_{17} 19_2 − 14 $2^p b_1$.
2_{20} $\boxed{\text{M01/1}}$
$1_{30}(2_7$ $17_{2,3}$ − 22)(2mc) $2^{2p}(a_2b_1+a_1b_2)$
↓
1_{31} Link

S.R.2

3_{28} 13 − 1_{31}
3_0 3_3 − 13 [3 3-13 5 6]
2_8 19_3 − 16 $2^p a_1$
2_{13} 19_3 − 14 $2^p a_1$
2_{20} $\boxed{\text{M01/1}}$
$1_{30}(3_3$ 3_5 − 13)[1 20-22 2 0 31]
3_6 19_2 − 16 $2^p b_1$
2_{14} 21 − 20 (2mc) $2^{2p} a_1^2$
2_{17} 19_2 − 14 $2^p b_1$
2_{20} $\boxed{\text{M01/1}}$
$1_{30}(3_5$ 20 − 22)(2mc) $2^{2p}(a_1^2+b_1^2)$
↓
1_{31}

FLOW DIAGRAM FOR DEUCE SUBROUTINE
NO. 123 (A08C) Complex multiply & modulus.

Date
File Ref. N.P.L. 66
Sheet Ref. 56/10408

DIVIDE

3_{23} $21 - 17_{2,3}$ (2mc)

3_{30} $13 - 1_0$

3_7 $3_{24} - 13$ [3 30-13 0 0]

3_{28} $\boxed{\text{S.R.2}}$

$1_{31}(3_{24}$ $30 - 13)$

3_1 $19_2 - 26$

3_4 $21 - 22$ (64-2p.mc) $2^{p+32}(a_1^2+b_1^2)$

3_{29} $13 - 19_2$ $-2^p b_1$

2_4 $21 - 17_{0,1}$ (2 mc.)

2_9 $3_{12} - 13$ [3 20-17 2 1 8]

3_{14} $17_{2,3} - 21$ (2 mc.)

2_{19} $\boxed{\text{S.R.1}}$

$1_{31}(3_{12}$ $20 - 17_{2,3})$ (2mc) $2^{2p}(a_1a_2+b_1b_2)$

3_9 $17_{0,1} - 20$ (2 mc) $2^{p+32}(a_1^2+b_1^2)$

3_{13} $3_{15} - 14$ [3 17-21 2 2 10]

3_{22} $30 - 13$

6_{28} $\boxed{\text{D. 13}}$

$1_{30}(3_{15}$ $17_{2,3} - 21)$ (2mc) $2^{2p}(a_1a_2+b_1b_2)$

3_{10} $13 - 17_0$ $2^p \dfrac{(a_1b_2-a_2b_1)}{a_1^2+b_1^2}$

3_{16} $3_{21} - 14$ [3 13-21 1 17]

3_{22} $30 - 13$

6_{28} $\boxed{\text{D 13}}$

$1_{30}(3_{21}$ $13 - 21_3)$ $2^p \dfrac{a_1a_2+b_1b_2}{a_1^2+b_1^2}$

3_{17} $17_0 - 21_2$ $2^p \dfrac{a_1b_2-a_2b_1}{a_1^2+b_1^2}$

↓

1_0 Link

SQUARE ROOT

3_{26} $13 - 1_1$

2_{18} $3_{25} - 13$

3_{27} $\boxed{\text{MODULUS}}$

1_0 $19_3 - 27$ $2^p a_1$

+ : 4_3 $19_3 - 25$ − : 4_4 $19_3 - 26$ Form $2^p\left\{|a_1|+\sqrt{a_1^2+b_1^2}\right\}$

4_8 $30 - 21_2$

4_{10} $13 - 21_3$ $\Big\}$ $2^{32+p}\left\{|a_1|+\sqrt{a_1^2+b_1^2}\right\}$ in 21

4_{14} $22 - 21$ (66-2pmc) $2^{2p}\left\{\dfrac{|a_1|+\sqrt{a_1^2+b_1^2}}{2}\right\}$

4_9 $4_{13} - 13$ [4 30-20 0 0]

5_{14} $\boxed{\text{F. 2}}$

$1_{30}(4_{13}$ $30 - 20_2)$

4_0 $30 - 21_2$

4_2 $13 - 20_3$ $2^{32+p}\left[\dfrac{|a_1|+\sqrt{a_1^2+b_1^2}}{2}\right]^{\frac{1}{2}}$ in 20

4_7 $16 - 21_3$ $2^{32+p} b_1$ in 21

4_{16} $22 - 21$ (66-2p mc) $2^{2p}\dfrac{b}{2}$ in 21

4_{11} $4_{15} - 14$ [4 20-15 1 1]

3_{22} $30 - 13$

6_{28} $\boxed{\text{D. 13}}$

$1_{30}(4_{15}$ $20_3 - 15)$ $2^p\left[\dfrac{|a_1|+\sqrt{a_1^2+b_1^2}}{2}\right]^{\frac{1}{2}}$

4_1 $19_3 - 27$ $2^p a_1$

+ : 4_5 $13 - 21_2$ − : 4_6 $13 - 21_3$

4_{12} $15 - 21_3$ 4_{17} $15 - 21_2$

1_1 Link

FLOW DIAGRAM FOR DEUCE SUBROUTINE NO. 123 (A08C) Complex divide and square root.	Date File Ref. N.P.L. 66 Sheet Ref. S6/10409

NELSON RESEARCH LABORATORIES, STAFFORD

THE ENGLISH ELECTRIC CO. LTD.

Date

Sheet No. [illegible]

Deuce Programme

CODING FOR DEUCE SUBROUTINE NO. 123 (ADRC)

D.L. 2 Track

Card Nos.

mc	NIS	S	D	C	W	T
0	2	2	13		9	17
1	2	2	13		1	10
2	2	2	13		3	10
3	2	15	16		1	1
4	2	21	17	2	2	3
5	2	20	21	2	0	4
6	2	2	13		2	12
7	1	17	22	2	2	31
8	2	21	16		0	2
9	3	3	13		1	3
10	2	20	23	2	0	2
11	3	21	22	1	2p-33	30
12	2	2	13		0	1
13	2	19	14		0	5
14	2	21	20	2	0	1
15	2	21	15		0	0
16	1	15	21		0	14
17	2	19	14		1	1
18	3	3	13		5	7
19	2	13	1		10	19
20	2	30	21		0	1
21	2	30	13		0	1
22	2	14	13		0	0
23	2	14	21		0	0
24	2	14	27		0	3
25	2	0	24		0	1
26	2	13	1		4	4
27	1	13	23		0	1
28	2	13	1		0	1
29	2	0	0		0	28
30	2	16	25		0	27
31	2	16	27		0	20

D.L. 3 Track

Card Nos.

mc	NIS	S	D	C	W	T
0	3	3	13		1	6
1	3	19	26		1	1
2	2	21	15		1	1
3	3	3	13		5	6
4	3	21	22	1	2p-40	23
5	1	[illegible]	22	2	0	31
6	2	19	16		0	6
7	3	3	13		15	19
8	2	19	16		1	3
9	3	17	20	2	1	2
10	3	13	17		0	4
11	2	29	23		1	3
12	3	20	17	2	1	8
13	3	3	14		0	7
14	2	17	21	2	2	3
15	3	17	21	2	2	10
16	3	3	14		3	4
17	1	17	21		1	13
18	3	3	13		0	8
19	1	13	13		0	0
20	5	3	13		18	3
21	3	13	21		1	17
22	6	30	13		0	4
23	3	21	17	2	1	5
24	3	30	13		0	0
25	4	19	27		1	1
26	2	13	1		5	22
27	3	13	1		3	21
28	3	13	1		1	2
29	2	13	19		1	5
30	3	13	1		0	7
31	3	29	23		1	1

D.L. 4 Track

Card Nos.

mc	NIS	S	D	C	W	T	
							X
							0
							1
0	4	30	21		0	0	2
1	4	19	27		0	2	3
2	4	13	20		1	3	4
3	4	19	25		0	3	5
4	4	19	26		1	2	6
5	4	13	21		1	5	7
6	4	13	21		1	9	8
7	4	16	21		0	7	9
8	4	30	21		0	0	Y
9	5	4	13		2	25	X
10	4	13	21		1	2	0
11	3	4	14		2	9	1
12	1	15	21		1	19	2
13	4	30	20		0	0	3
14	4	22	21	1	2p-40	25	4
15	4	20	15		1	1	5
16	4	22	21	1	2p-40	25	6
17	1	15	21		1	14	7
18							8
19							9
20							Y
21							X
22							0
23							1
24							2
25							3
26							4
27							5
28							6
29							7
30							8
31							9